THE INTELLECTUAL HERITAGE OF BABYLONIAN ASTRONOMY: MUSIC OF THE SPHERES

Immanuel Freedman, Ph. D CPhys MInstP SMIEEE

Freedman Patent

LEARNING OBJECTIVES

- Understand the possible role of cultural contact between Scribes and Pythagoreans
- Understand relations between music and Babylonian astronomy
- Explain the zodiacal distribution of Babylonian Normal Stars
- Understand relations between the mathematics of Babylonian astronomy and a modern chaos theory of nonlinear dynamics
  – Chaos theory describes systems highly sensitive to initial conditions
- Apply mathematics of Babylonian astronomy to a modern problem

CONSTRUCTION OF PYTHAGOREAN MUSICAL SCALES

Unlimited (ἄπειρον) continuum of pitches limited (πέρας) by intervals interacting according to a harmony (Ἁρμονία)

Octave: 2/1
Fifth: 3/2
Fourth: 4/3
Remainder (λεῖμμα) : 243/256

Fragments 1, 5, 6, 6a, 7
(Philolaus, c. 470-c. 385 BCE)
Stanford Encyclopedia of Philosophy: Philolaus

Faster speed means higher pitch

"…there is no [rational] mean proportional between numbers in superparticular ratio [(n+1)/n] and hence the basic musical intervals cannot be divided in half."

Fragments A19, B1
(Archytas, c. 428-c. 347 BCE)
Stanford Encyclopedia of Philosophy: Archytas

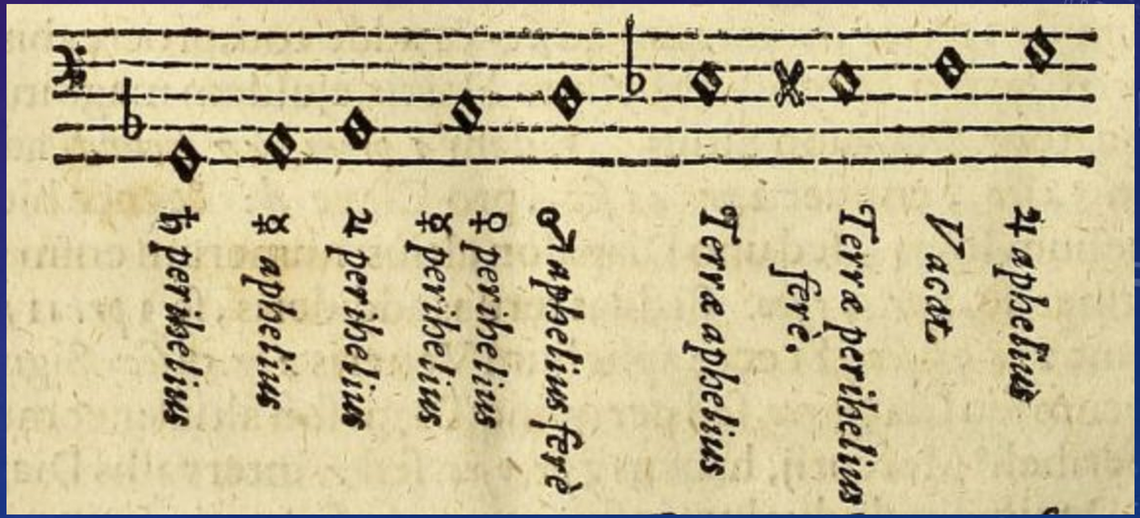
COSMOLOGY: MUSIC OF THE SPHERES

"World Soul" musical scale based on Pythagorean intervals:

Unison:	1/1
Perfect fourth:	4/3
Perfect fifth:	3/2
Tone:	9/8
Diapason:	2/1

Timaeus
(Plato, c. 360 BCE)

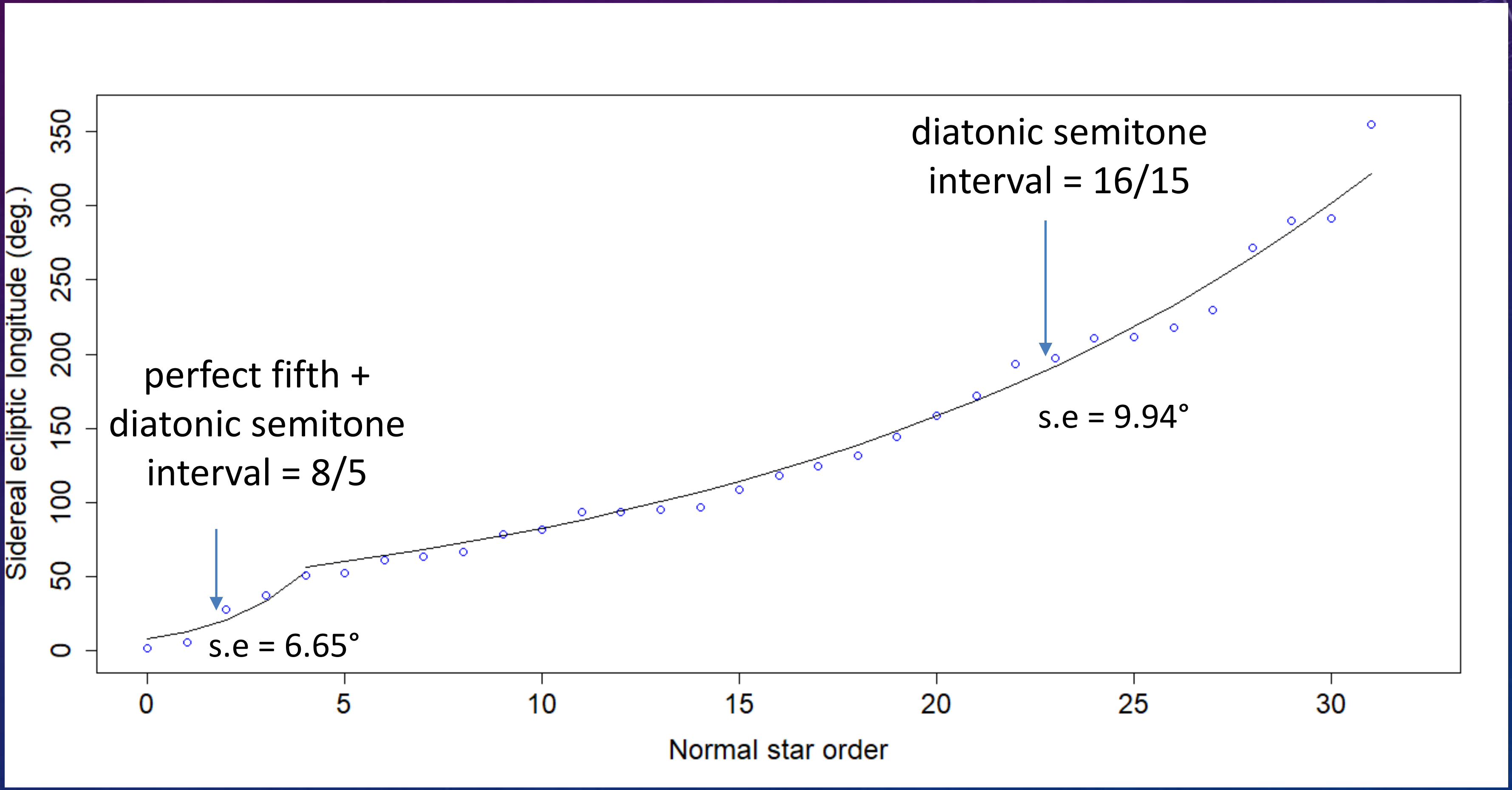
Musical scale based on planetary angular speed when closest to or furthest from Sun:



De Harmonices Mundi, Lib. V, Cap. V
(Kepler, 1699 CE)

A MUSICAL SCALE OF STARS

MULŠIDMEŠ=*kakkabū minâti* (counting stars)



Data from Hunger, H. & Pingree, D. E. "Astral Sciences in Mesopotamia," p. 48

MUSICAL HARMONICS ARE RESONANCES

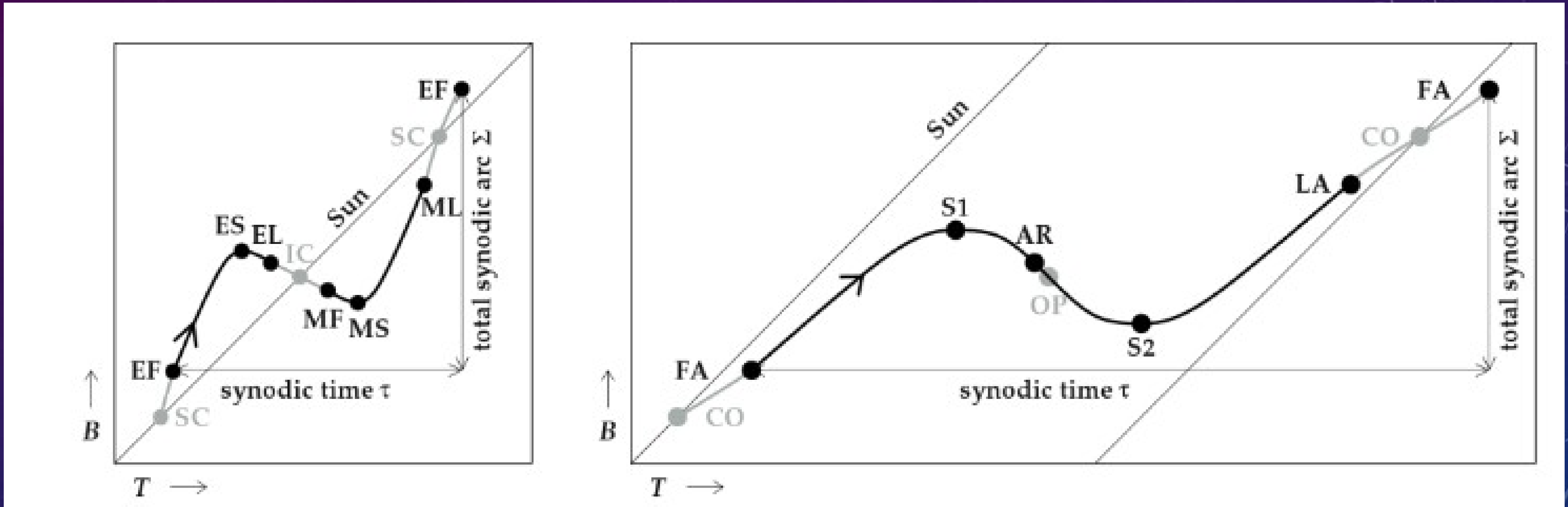
- Picture shows Queen’s lyre reconstruction

- Strongest and most stable resonances have
  - superparticular ratio of string length: (n±1)/n
  - small whole number denominator

British Museum # 121198,a c. 2600 B. C. E.
Waltham, C. (2008), J. Acoust Soc Am 123, 3661
Reichenbach, T. & Hudspeth, J. A. (2014), ArXiv/1408.2085
Lots, I. S. & Stone, L. (2008), J. R. Soc. Interface 5, 1429-1434
Heimpel, W. & Szabo, G. “Strings and Threads”, references therein

HYPOTHESIS: PLANETARY EVENTS ARE WELL-DEFINED STATES

ZODIACAL POSITION (B) vs. TIME (T)



Inner planet events

Outer planet events

SC=Superior Conjunction, EF=Evening First, ES=Evening Station, EL=Evening Last, IC = Inferior Conjunction, ML = Morning Last; CO = Conjunction, FA = First Appearance, S1=First Station, AR=Acronychal Rising, S2 = Second Station, LA = Last Appearance

Ossendrijver, M. (2012), Babylonian Mathematical Astronomy: Procedure Texts, p. 56

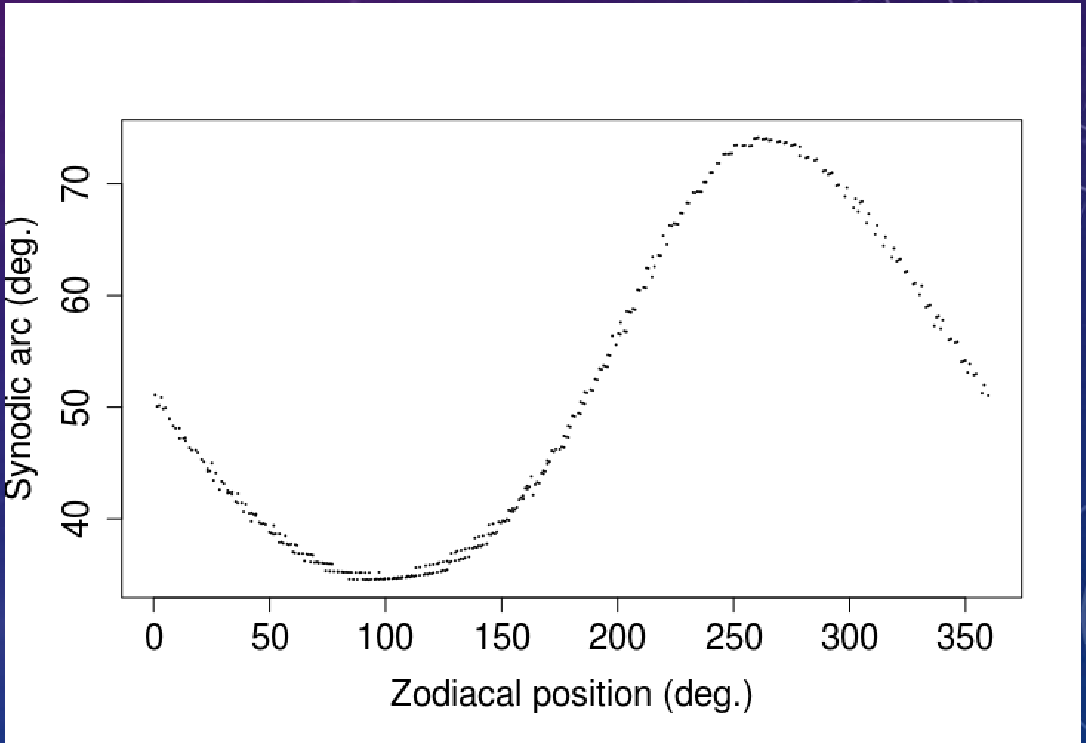
IDEAL SIMULATION OF MARS FIRST APPEARANCE

FIRST RETURN MAP: 15° SOLAR ELONGATION

Start:	1/1/747 BCE	
End:	1/1/76 CE	
Location:	Babylon	
Elevation:	91 m	(E_2-TEMEN-AN-KI)
Equinox:	Hamal	

Empirical rotation/event:

Average:	48.76°
Minimum:	34.58°
Maximum:	74.10°

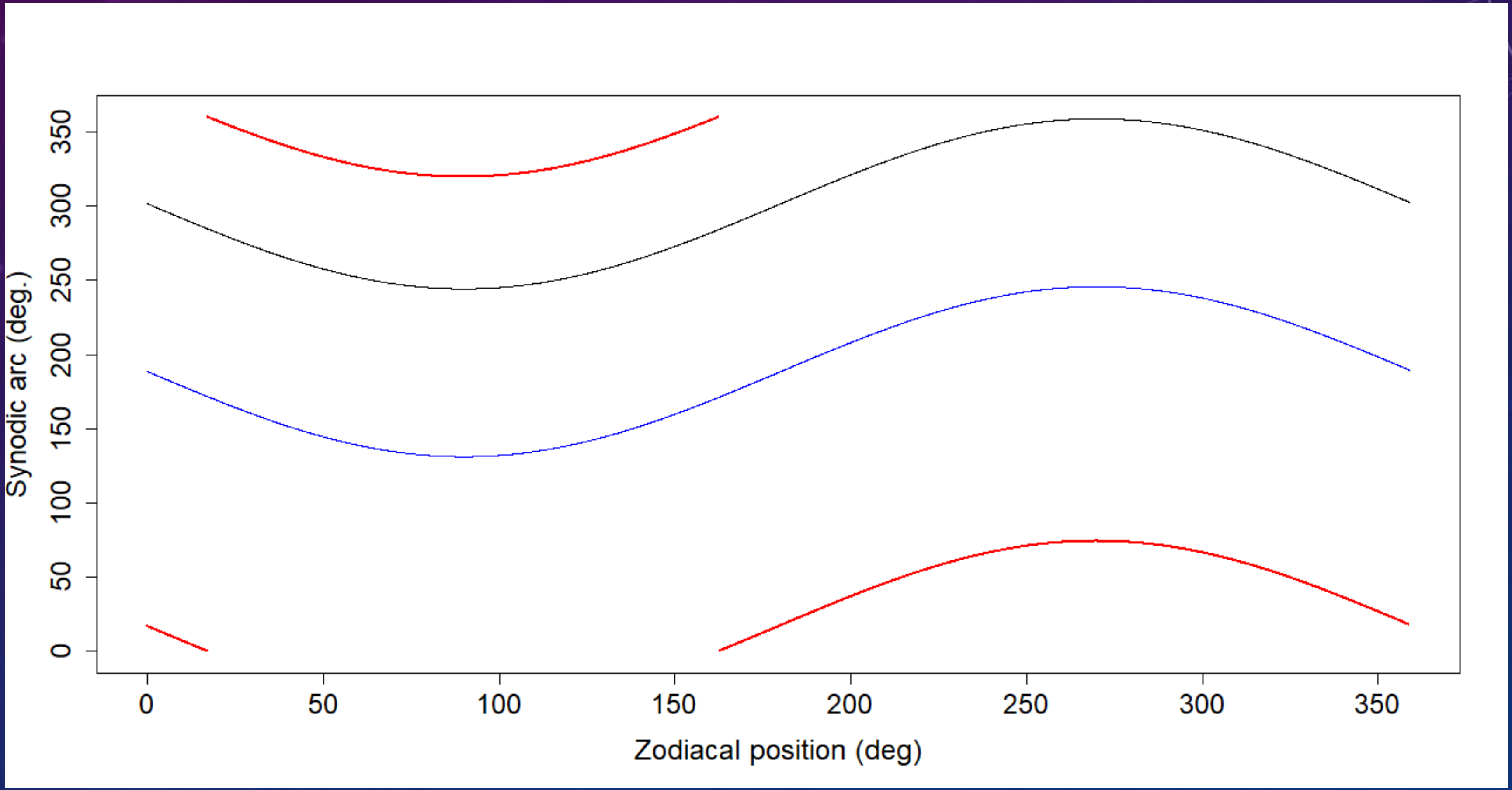


Forced oscillation:

The invariant curve is approximately a thin line

Modeling software includes Python/PyEphem, R/Rstudio

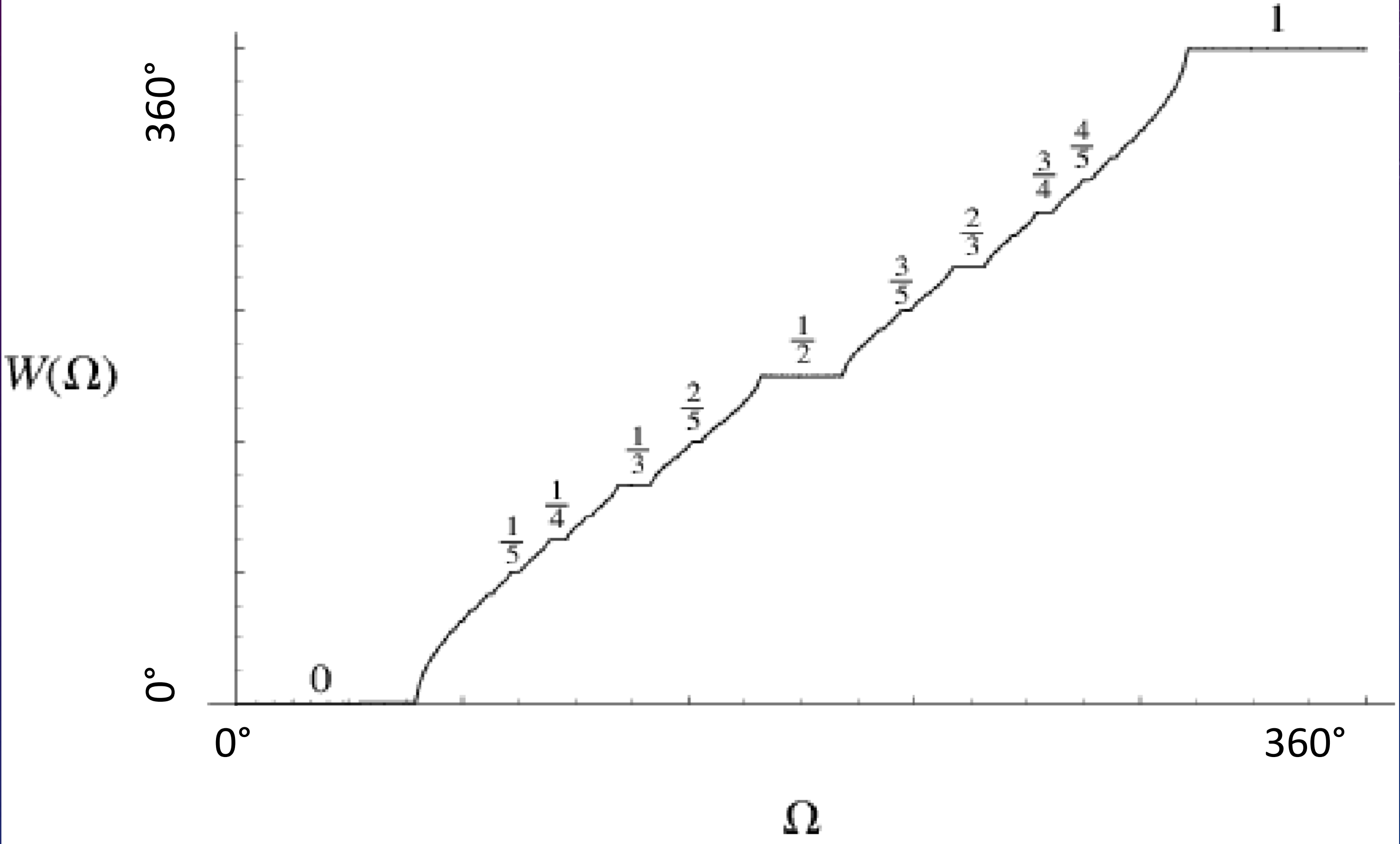
EXEMPLARY FAMILY OF SINE CIRCLE MAPS
FORCED OSCILLATIONS AT THE BORDER OF CHAOS



The maps wrap around the circle (0° to 360°)

Oscillator fundamental rate of rotation: 30°/event (blue), 48°/event (black), 60°/event (red)

SINE CIRCLE MAP RESONANCE: SYNCHRONIZATION
FORCED OSCILLATIONS AT THE BORDER OF CHAOS

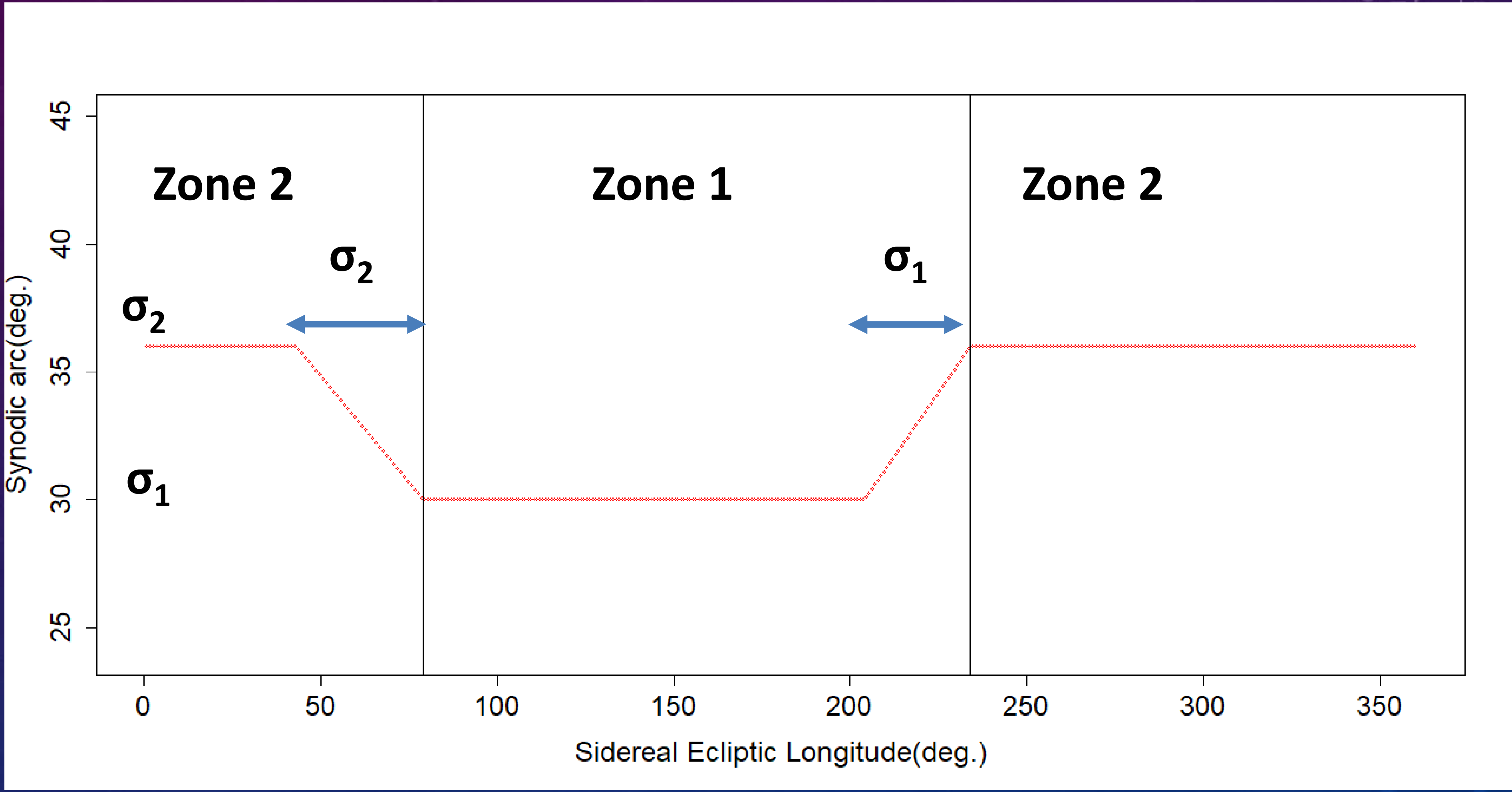


W(Ω): asymptotic average rate of rotation per event

Ω: fundamental rate of rotation per event

Retrieved from https://en.wikipedia.org/wiki/Arnold_tongue

EXEMPLARY 'SYSTEM A' MAP



'System A' maps are subject to periodicity constraints
All quantities are rational numbers that terminate in base 60 to avoid rounding

'SYSTEM A': MUSICAL MOTIVATION FOR ANGULAR RATE

- The planetary events move according to 'elementary steps' in each zone
- Larger 'elementary steps' imply faster angular rate
- Faster angular rate implies higher pitch (Archytas, *op*. *cit*.)
- Pitches interact according to a harmony (Philolaus, *op*. *cit*.)
- The strongest pitch interactions are musical resonances
- There is a cosmic harmony (Plato, *op*. *cit*.)
- The basic musical intervals cannot be divided in half (Archytas, *op. cit.)*

Hence, 'System A' map zones should be chosen according to a musical scale

MUSICAL INTERVALS OF ANGULAR RATE (1 of 2)

‘SYSTEM A’ PLANETARY MODELS

Object	System	Elementary Step (°)	Interval Ratios	Interval Names
Sun	A	1/8	1/1, 16/15	unison, major semitone
Mercury	$A_1(\Gamma)$	1/9	9/8, 3/2, 1/1	major tone, perfect fifth, unison
	$A_1(\Xi)$	2/9	3/2, 1/1, 9/10	perfect fifth, unison, minor tone
	$A_2(\Sigma)$	1/3	5/6, 1/1, 3/4, 1/1	minor third, unison, perfect fourth, unison
	$A_2(\Omega)$	5/8	4/5, 8/9, 4/5, 1/1	major third, major tone, perfect fifth, unison
Mars	A	5/2	1/1, 2/3,8/9,4/3, 2/1, 3/2	unison, perfect fifth, major tone, perfect fourth, diapason, perfect fifth

Γ=morning rising, Ξ = evening rising, Σ=morning setting, Ω=evening setting

Subharmonics suggest nonlinearity

Data from Aaboe, A. (1964), Centaurus 10, 221

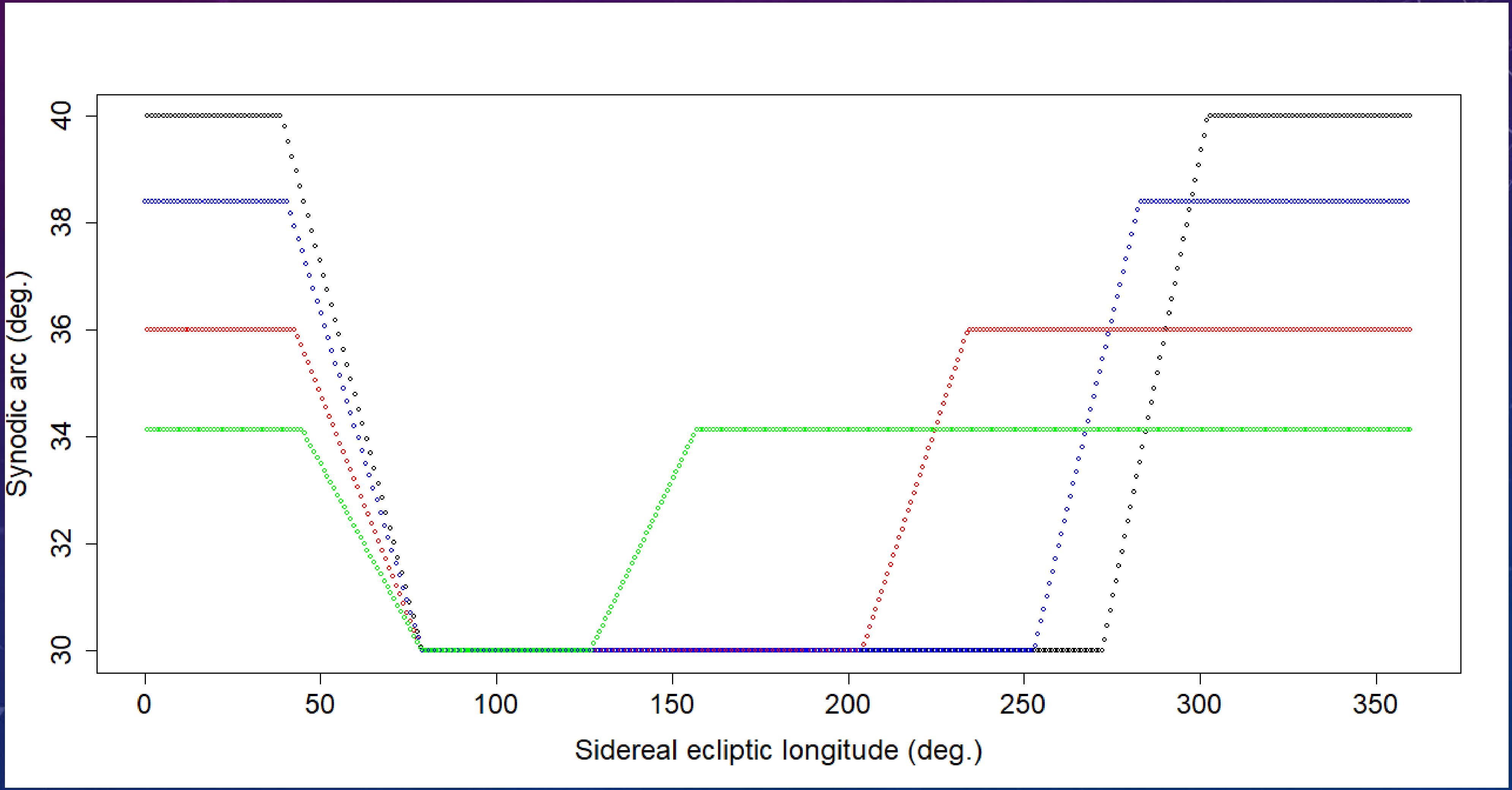
MUSICAL INTERVALS OF ANGULAR RATE (2 of 2)

'SYSTEM A' PLANETARY MODELS

Object	System	Elementary Step (°)	Interval Ratios	Interval Names
Jupiter	A	5/6	6/5, 1/1	minor third, unison
	A_1	1/3	6/5, 1/1	minor third, unison
	A'	1	5/6, 15/16, 1/1, 15/16	minor third, major semitone, unison, major semitone
	A''	1	5/6, 15/16, 1/1, 15/16	minor third, major semitone, unison, major semitone
	A'''	4/3	5/6, 15/16, 1/1, 15/16	minor third, major semitone, unison, major semitone
Saturn	A	25/16	5/6, 1/1	minor third, unison

Data from Aaboe, A. (1964), Centaurus 10, 221

FAMILY OF 'SYSTEM A' MAPS WITH EQUAL PERIOD
IDEAL JUPITER FIRST APPEARANCE



EXEMPLARY OPTIMIZATION OF MAP PARAMETERS

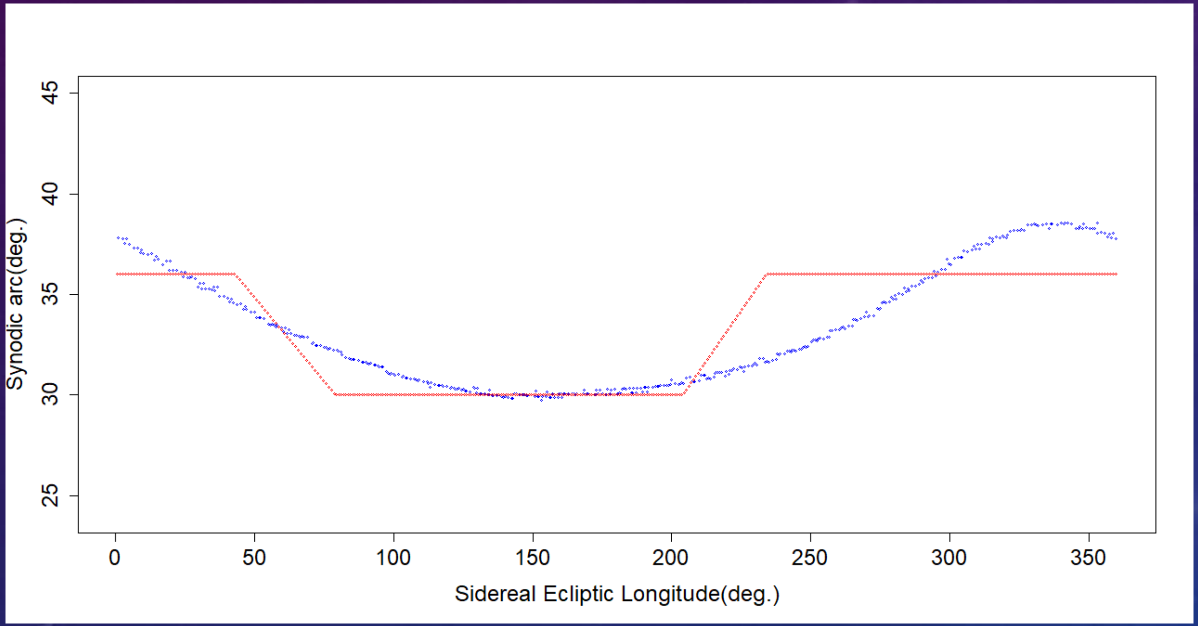
HYPOTHESIS: CLOSELY MATCHING THE SEQUENCE OF ZODIACAL SIGNS

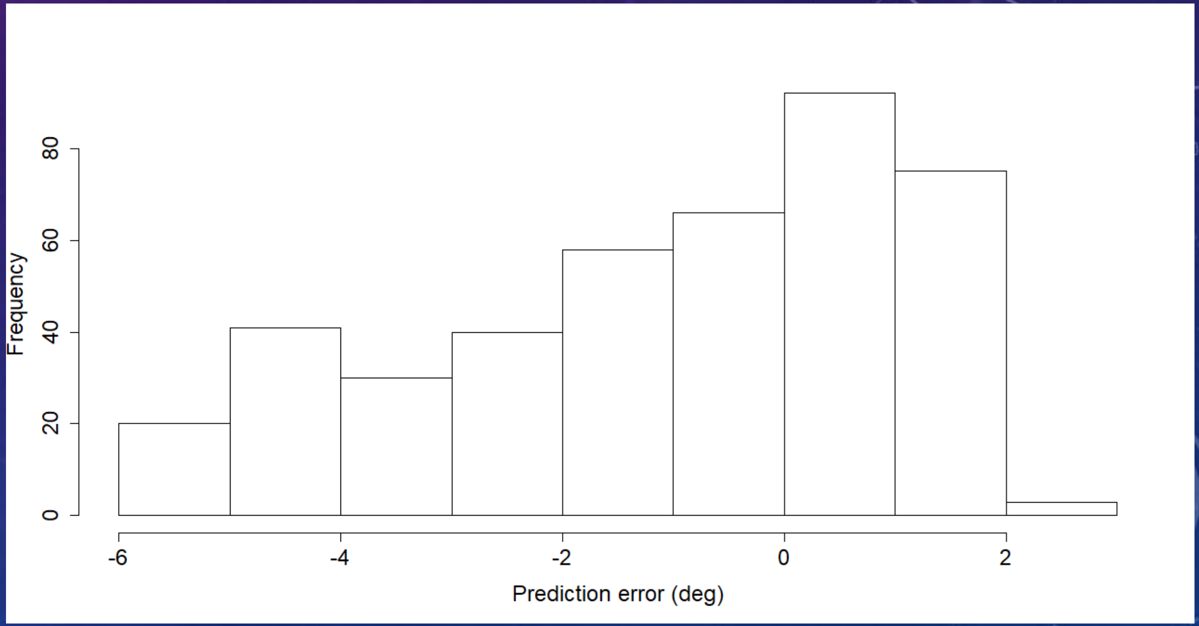
Observations:	26 Leo	01 Vir	10 Lib	1 Sag	13 Aqu
Predictions:	20 Leo	29 Leo	05 Sco	3 Sag	16 Aqu

Score: 1/5 wrong signs

- The map is chosen to minimize the number of wrongly-predicted signs
- Signs are matched in the computational zodiac with 12 signs of 30°
- Planets on the cusp (within a 5° orb) are considered in the same sign

‘SYSTEM A’ MAP (OPTIMAL)
IDEAL JUPITER FIRST APPEARANCE

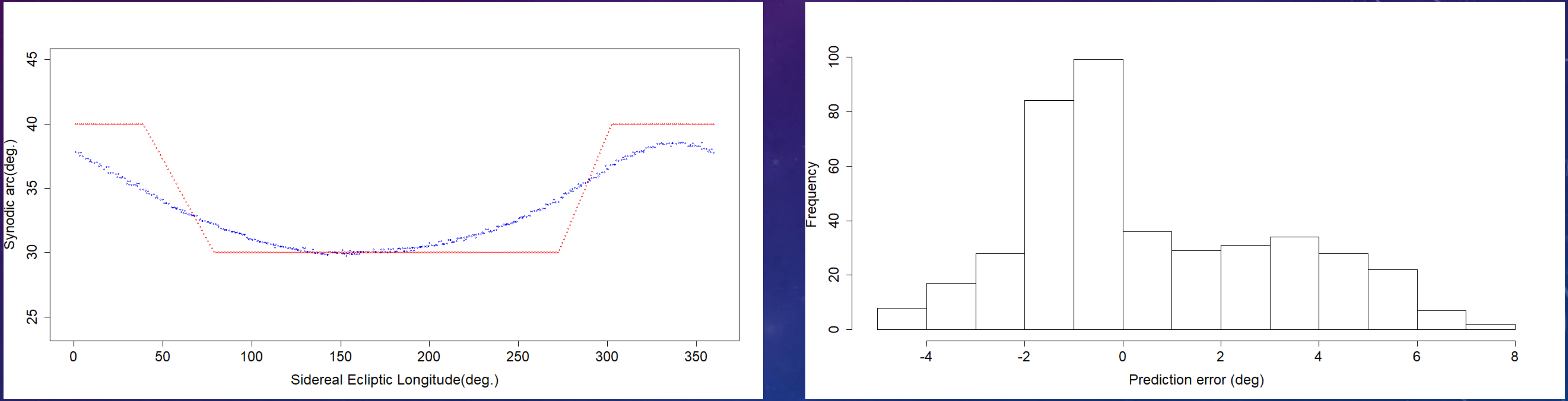




‘System A’ map (red), simulated observations (blue), score = 9/424

σ_1 set to 30°/event; optimized σ_2= 36°/event from choice of 36° (6/5) or 40° (4/3) /event

‘SYSTEM A’ MAP (NOT OPTIMAL)
IDEAL JUPITER FIRST APPEARANCE



‘System A’ map (red), simulated observations (blue), score = 19/424

σ_1 set to 30°/event, σ_2 set to 40°/event

CARDIAC ARRHYTHMIA

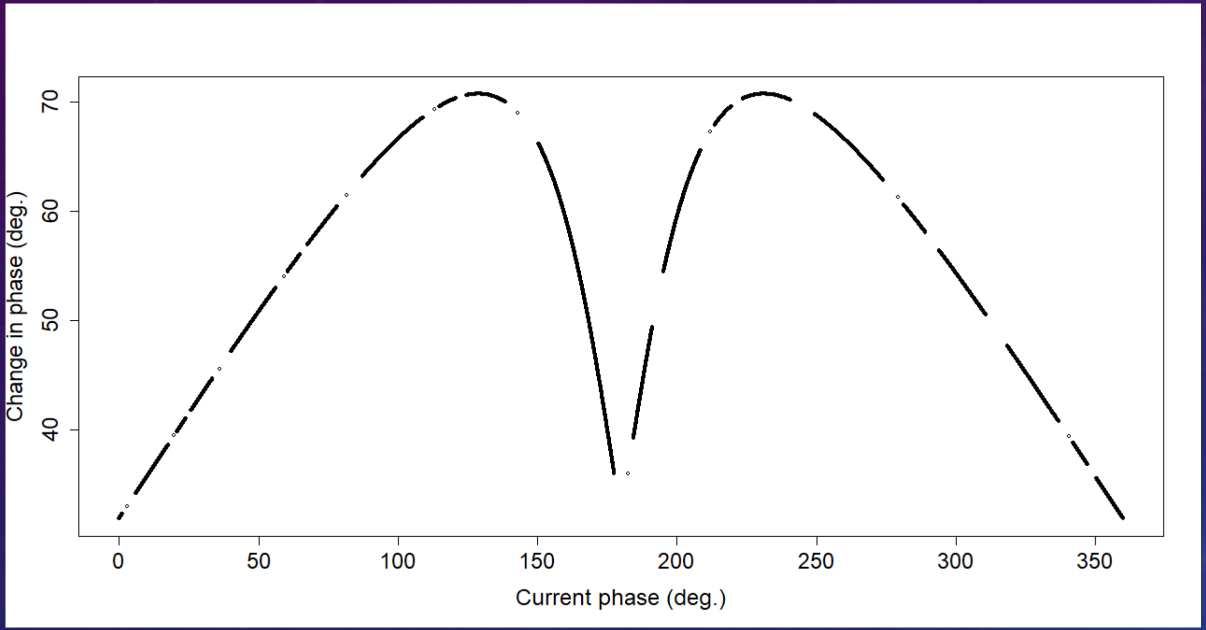
VENTRICULAR PARASYSTOLE

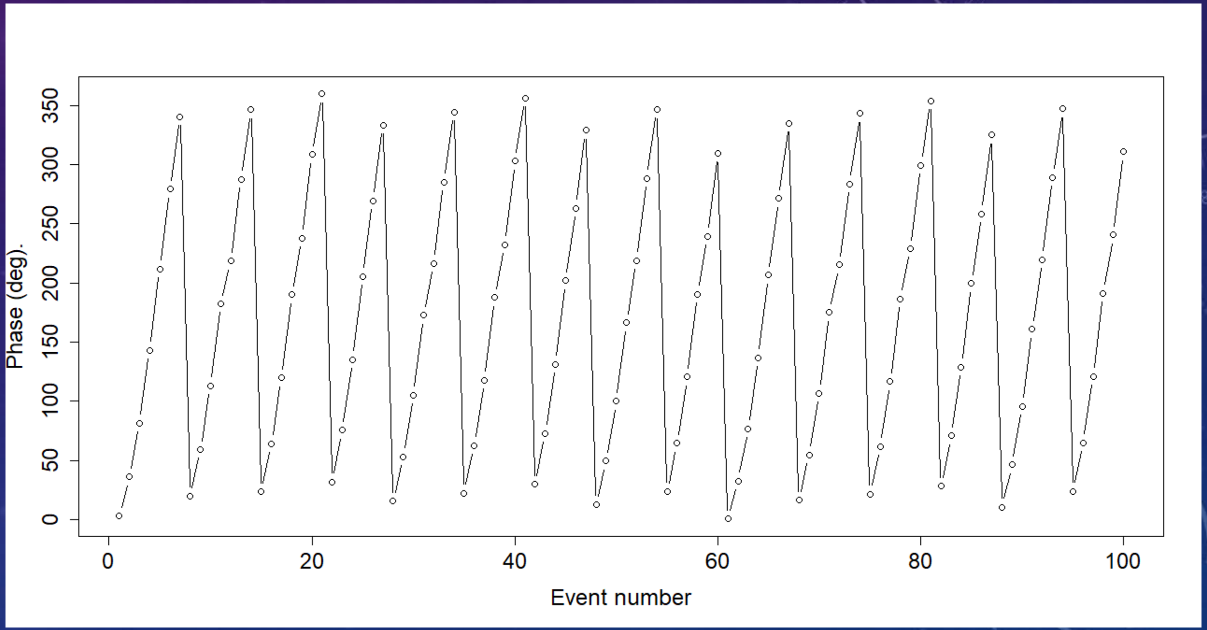
S (Sinus) and E (Ectopic) denote natural pacemakers

Glass, L. (1991), Chaos 1,13

Retrieved from https://ecg.utah.edu/

SIMULATED CARDIAC ARRYTHMIA
FIRST RETURN MAP AND TIME SERIES



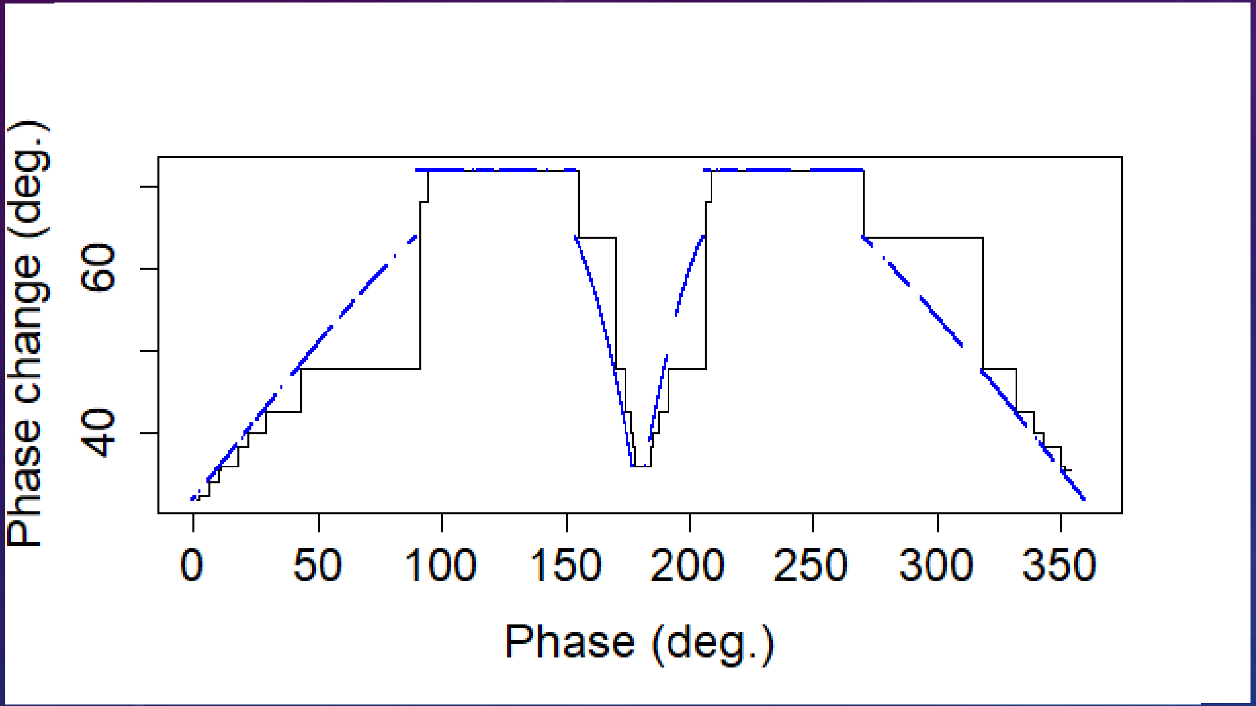


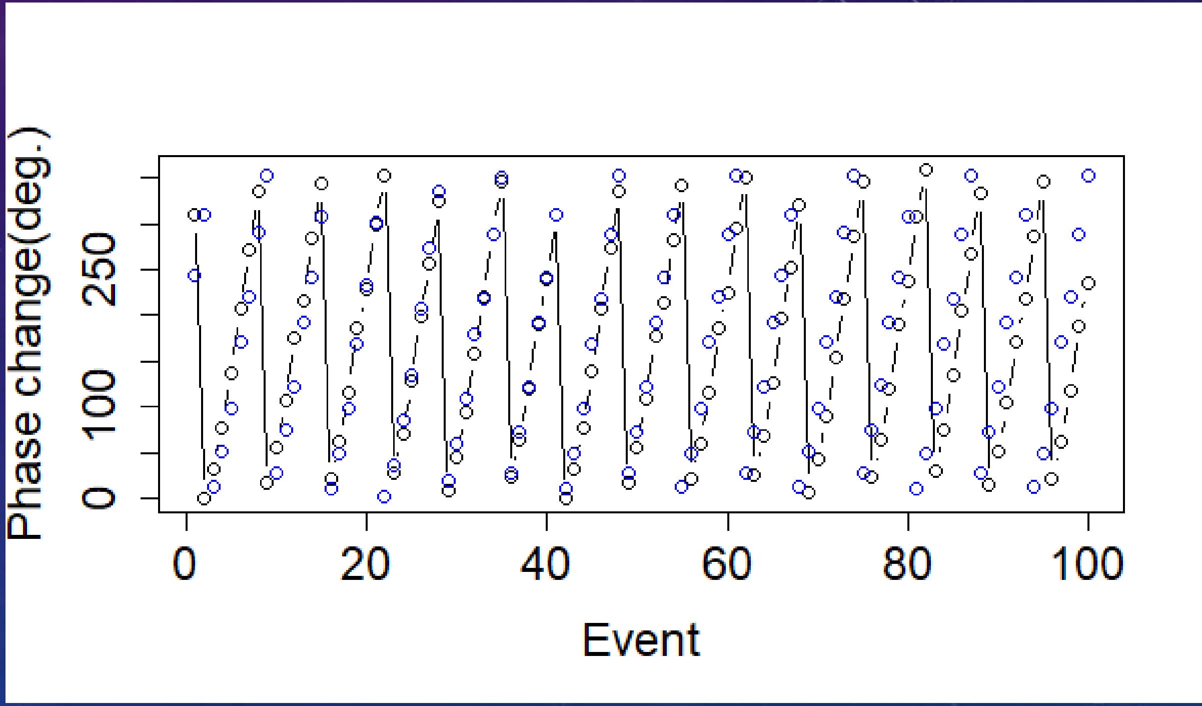
Sinus: 72 bpm, Ectopic: 40 bpm (63% coupling)

Glass, L. (1991), Chaos 1,13

Hoppensteadt, F. & Keener. J. (1982), J Math Biol, 339-349

SIMULATED CARDIAC ARRYTHMIA
‘SYSTEM A’ APPROXIMATION





Fundamental rate of rotation per event: 32°, period = 20/3 events

Resonant levels: 1/1, ,81/80, 16/15, 10/9, 9/8, 6/5, 5/4, 4/3, 3/2, 32/15, 9/4

SUMMARY & IMPLICATIONS

- Cultural interaction between Scribes and Pythagoreans may have inspired the choice of superparticular ratios in Babylonian astronomy
- The forced oscillations of ‘System A’ maps are well described in terms of musical resonance
- The mathematical methods of Babylonian astronomy are still useful

QUESTIONS?

BACKUP SLIDES

SYSTEM A PERIODICITY CONSTRAINTS

System A models with transition rules are constrained as follows:

$$\sum_{i=1}^{n} \frac{\alpha_i}{w_i} = \frac{\Pi}{Z}, \sum_{i=1}^{n} \alpha_i = 6{,}0°$$

where w_i, α_i denote the angular frequency and width of each of n zones, Π denotes the asymptotic average number of synodic events of a particular type in Z rotations of the event through the ecliptic.

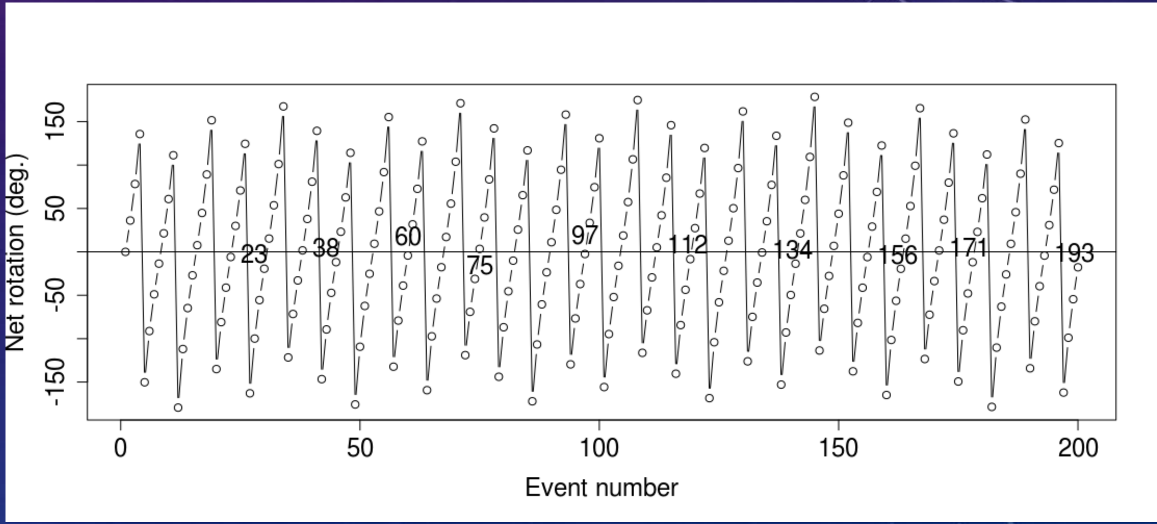
All the quantities are rationals with denominators divisible by 2, 3, or 5 to avoid rounding error in base 60.

Swerdlow, N.M, "The Babylonian Theory of the Planets"

IDEAL SIMULATED MARS FIRST APPEARANCE

REFINING THE PERIOD BY GOAL-YEAR ANALYSIS

- Assume *each observation is representative of the whole*
  - haeccity, ergodicity
- Consider close passages by Babylonian Normal Stars
- Combine return periods to offset net rotation



Long period of 47+3*79 = 284 years yields net rotation (-6°)+3*(2°) ~ 0° (actually -1°)

Neugebauer, O., *Astronomical Cuneiform Texts* II, p. 302
Duns Scotus (1266-1308 CE); *Ordinatio* II, d. 3, p. 1, qq. 5–6, n. 187

CONSTRAINTS ON THE AVERAGE SYNODIC PERIOD

- Let R bound the average period of rotation through the ecliptic from above or below by at most 1y
- To a precision of about 0.01 y, a rational approximation to the period with denominator divisible by 2, 3, or 5 is given either of the following formulae

$$P \equiv \frac{\Pi}{Z} = R - \left[\frac{1}{r} + \frac{1}{r \cdot s}\right];\ r, s \in \{1, 2, 3, 4, 5, 6, 8, 10\} \text{ (Equation 1)}$$

$$P \equiv \frac{\Pi}{Z} = R + \left[\frac{1}{r} + \frac{1}{r \cdot s}\right];\ r, s \in \{1, 2, 3, 4, 5, 6, 8, 10\} \text{ (Equation 2)}$$

Either of these expansions in Egyptian fractions may efficiently enumerate all the feasible periods with only 16 alternatives if R is known

Aaboe, A. (1964), Centaurus 10, pp. 213-321

SELECTION OF SYNODIC PERIOD

- Initial estimate of period based on average synodic arc
  - $\bar{\sigma}$=360/48.76 = 7.38 events, bounded by 7 from below.
- Expansion in Egyptian fractions (Equation 2, Engel expansion)
  - Initial bound: 7
  - First approximation: 7 +(1/3) *r*=3
  - Second approximation: 7+(1/3) +(1/18) *r*=3, *s*=6
- Period P=2,13/18 approximates the average synodic arc to better than 0.1%.
  - The result , Π=133 synodic events and Z=18 rotations through the ecliptic in 284 sidereal years, is consistent with *ACT* 811, 811a and the simulation.

MODULATED PARASYSTOLE CIRCLE MAP

A stable clock of unit period and amplitude whose phase is reset by periodic impulses of period T and amplitude A projected on the horizontal axis has phase map given by

$$\theta_{n+1} = \theta_n + T + \cos^{-1}\left(\frac{1 + A \cdot \cos\theta_n}{\sqrt{1 + 2A \cdot \cos\theta_n + A^2}}\right)$$

Hoppensteadt, F. & Keener. J. (1982), J Math Biol, 339-349

MODERN COSMOLOGY IS STILL PYTHAGOREAN

SYMMETRIES OF THE UNIVERSE

Tetraktys of fundamental particles

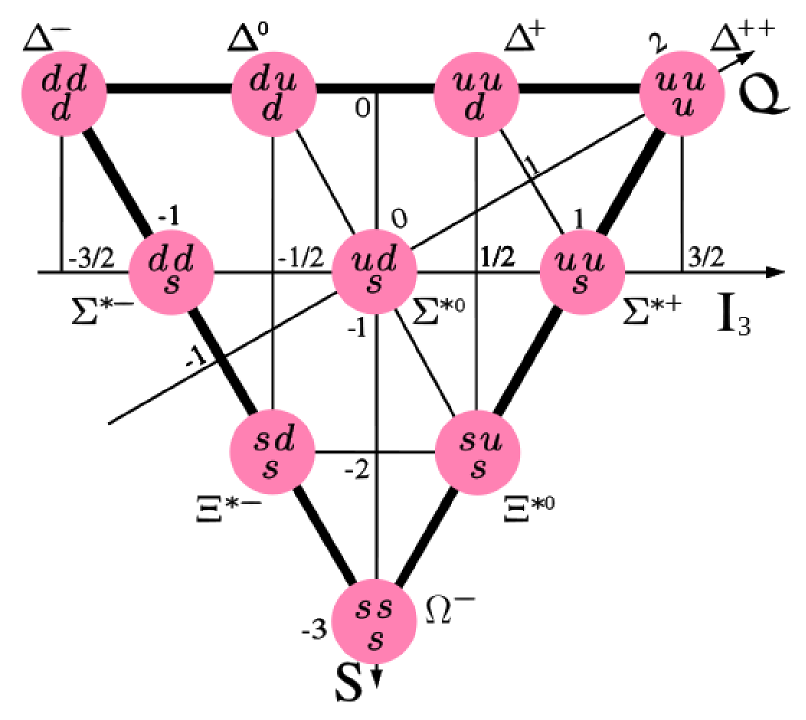
The baryons may be composed of up (*u*), down (*d*) or, strange (*s*) quarks

Quantum numbers:

Q: electric charge

I_3: isospin

S: strangeness



Gell-Mann, M. & Goldberger, M.L. Phys. Rev 96, 1964; 1433-8

Retrieved from https://en.wikipedia.org/wiki/Clebsch%E2%80%93Gordan_coefficients_for_SU(3)